Pythagoras' theorem

More Details

Ancient technique – current mathematics

In order to achieve the astonishing precision of their pyramids the Egyptian rope tensioners divided a long rope into twelve parts with knots. With the help of stakes, they formed a triangle with a ratio of 3:4:5. In this way, a right angle is always formed. Indian priests also used this method to determine right angles, for example when building their altars, but with the aspect ratio of 15:36:39.

What the Egyptians and Indians developed through practical application, received its mathematical presentation in Pythagoras' theorem with $a^2 + b^2 = c^2$ In fact, $3^2 + 4^2 = 5^2$ (Egyptian rope tensioners) and $15^2 + 36^2 = 39^2$ (Indian priests) are valid equations.

Pythagoras' theorem

The Pythagoras' theorem is one of the fundamental theorems of Euclidean geometry (geometry of the two- and three-dimensional space known to us after the Greek mathematician Euclid). Pythagoras' theorem states that in all flat, right-angled triangles, the sum of the cathetus squares a^2+b^2 is equal to the hypotenuse square c^2:

$a^2 + b^2 = c^2$

whereby *a* and *b* represent the lengths adjacent to the right angle, called legs or catheti, and *c* the length opposite of the right angle, the hypotenuse (Fig. 1).

The inversion also applies: if the equation $a^2 + b^2 = c^2$ applies in a flat triangle, then this triangle is right-angled, whereby the side with the length c is opposite the right angle.

A so-called Pythagorean triple is a group of three whole numbers for which the equation $a^2 + b^2 = c^2$ applies. There is an infinite number of triples with this property. The simplest triple is formed by the numbers (3,4,5) (see Egyptian rope tensioners), since $3^2 + 4^2 = 5^2$. Other triples are for example (6,8,10) or (15,20,25) and (15,36,39) (see Indian priests).

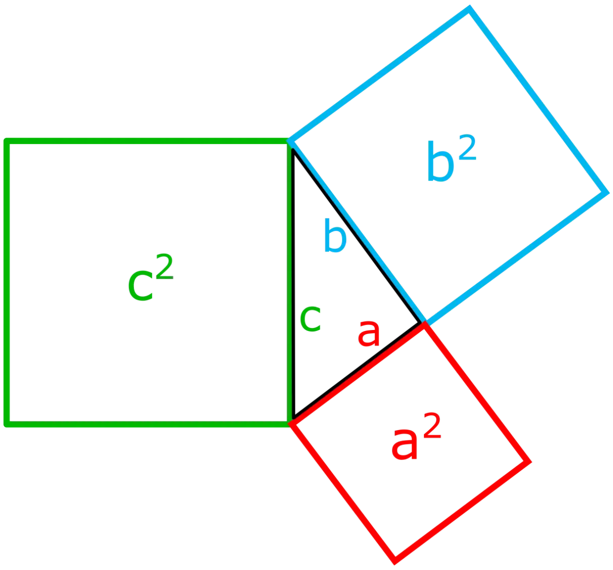

Fig. 1: Pythagoras' theorem $a^2 + b^2 = c^2$.

Geometric mean theorem

Pythagoras' theorem also includes the geometric mean theorem: The squared height of a flat, right-angled triangle is equal to the rectangle formed by the two hypotenuse sections.

Expressed as an equation, the geometric mean theorem is:

$h^2 = p \cdot q$

whereby h represents the height of the triangle on the hypotenuse c and p and q represent the two hypotenuse sections (Fig. 2).

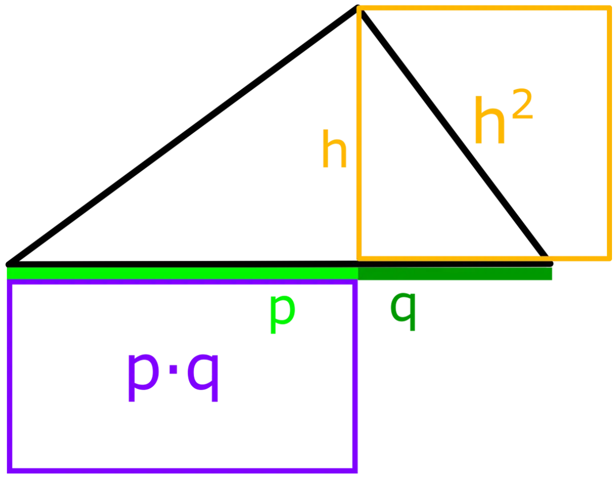

Fig. 2: Geometric mean theorem $h^2 = p \cdot q$.

Cathetus theorem

Another fundamental theorem is the cathetus theorem. It states that in a flat, right-angled triangle, the following applies: The area of a cathetus square is equal to the area of the rectangle which is formed from the length of the hypotenuse and the length of the associated hypotenuse section.

Expressed as an equation, the cathetus theorem is:

$a^2 = c \cdot q$ or $b^2 = c \cdot p$

whereby a or b represents the length of the cathetus, c represents the length of the hypotenuse, and q or p represents the length of the associated hypotenuse section (Fig. 3).

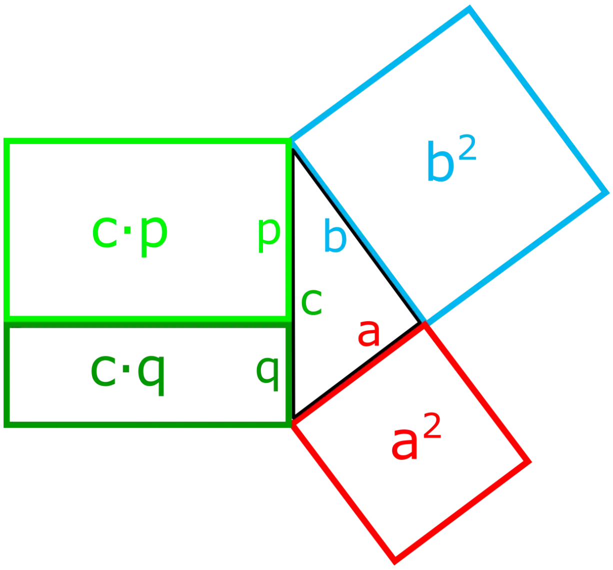

Fig. 3: Cathetus theorem $a^2 = c \cdot q$ or $b^2 = c \cdot p$.