Chaotic pendulum

More Details

Unpredictable pendulum

Our pendulum follows laws of nature that we know precisely. Thus, one should think that we can calculate its movements and predict exactly how it will move.

This does apply to very small deflections: the force that pulls a pendulum back into its resting position increases the further you deflect it. This relationship is linear. However, at larger deflection angles, it becomes non-linear, meaning that the force no longer increases to the same extent as the deflection. For example, when a pendulum is upside down (deflection of 180°), there is no force acting at all. However, slightly off, with a deflection of 179.99°, the force is very large.

Now we are not just dealing with a simple pendulum, but with four moving parts – making it even more complex. The non-linearity makes it impossible for us to predict how the pendulum will move in the next moment – one could say that it moves randomly.

But that is not the case! At every moment, it still follows the exact laws of nature that we know. Nevertheless, a very small difference in the beginning has a considerable impact. Since we usually do not notice this small difference, we think that the pendulum is moving randomly. However, it is not a random, but what scientists refer to as chaos.

Chaos – a law of nature

In the 17th century, Isaac Newton formulated his basic mechanical laws. Since then, it has long been thought that the world was precisely predictable. The scientist Laplace said in 1814: "An intelligence knowing all the forces acting in nature at a given instant, as well as the momentary positions of all things in the universe, would be able to comprehend in one single formula the motions of the largest bodies as well as the lightest atoms in the world, provided that its intellect were sufficiently powerful to subject all data to analysis; to it nothing would be uncertain, the future as well as the past would be present to its eyes."

He thought that the result of rolling the dice is only unpredictable because so many different things affect the result that we cannot measure and consider: unevenness of the floor, the smallest dents in the dice, air circulation, etc. However, Newton had already determined that he could calculate the movement of two celestial bodies, but not three – although he knew the calculation formula, he could not solve it!

Another famous scientist, Henri Poincaré, then proved in 1889 that no one, not even the best computer in the world, could solve this problem. So, there are actually systems that are unpredictable, even though we know the laws they follow. We call these systems chaotic.

Nevertheless, chaos is not completely inscrutable: chaos research (or better: non-linear dynamics) can make certain statements about the behavior of chaotic systems and therefore better understand the chaos that surrounds us.

Examples of chaotic systems

Many systems in our world are chaotic, for example the weather, our heartbeat, the brain, stock exchange prices, billiard. Even mixing milk into coffee is non-linear and therefore chaotic. And that is a good thing

– if things were different, we would have to stir for hours in order to have an actual milk coffee.

Small animal, big effect – the butterfly effect

When it comes to weather, people often speak of the so-called “butterfly effect”, which makes it impossible to forecast the weather weeks in advance. Referring to the metaphor, that a flap of a butterfly’s wings can influence the weather in a different place, the meteorologist Edward L. Lorenz illustrated that small differences in the state of the atmosphere can lead to different forecasts in the weather prediction models. These differences arise, among other things, from missing or incorrect measurements. Additionally, the equations in the weather prediction models have been simplified, so that they can be used for calculations in the first place. For the best possible weather forecasts, these uncertainties are taken into account in so-called ensemble forecasts. Weather prediction models start out with several slightly different initial conditions of the atmosphere and the results are compared with one another. For example, if all calculations show a temperature increase over the next few days, it will very likely become warmer, even if the exact temperature varies slightly between the individual calculations. Therefore, a weather forecast is reliable approximately seven days in advance. Seasonal forecasting, however, is not as reliable.

The butterfly effect does not only occur in weather, but in all chaotic systems, as well as in the chaotic pendulum. Small changes in the starting position of the pendulum (wing flap of the butterfly) cause different complex motion sequences (tornado).